5.1 Branch-and-Bound

Given the integer linear program

$$\begin{aligned} \max z = 3x_1 + 4x_2 & \\ 2x_1 + x_2 & \leq 6 \\ 2x_1 + 3x_2 & \leq 9 \\ x_1, x_2 & \geq 0 \text{ integer} \end{aligned}$$

solve it via the Branch-and-Bound method (solving graphically the continuous relaxation of each subproblem encountered in the enumeration tree). Branch on the fractional variable with fractional value closest to $\frac{1}{2}$. Among the set of active nodes, pick that with the most promising bound.

5.2 Branch-and-Bound for 0-1 knapsack

A bank has 14 million Euro, which can be invested into stocks of four companies (1, 2, 3, and 4). The table reports, for each company, the net revenue and the amount of money that must be invested into it.

Company	1	2	3	4
Revenue	16	22	12	8
Money	5	7	4	3

Give an Integer Linear Programming formulation for the problem of choosing a set of companies so as to maximize the total revenue. Note that no partial investment can be done, i.e., for each company we can either invest into it or not. Solve the problem with a Branch-and-Bound algorithm. Show that the continuous relaxation of the original problem and the resulting subproblems can be solved to optimality with a simple greedy algorithm.

5.3 Cutting plane algorithm

Given the integer linear program

$$\begin{aligned} \min \quad x_1 - 2x_2 & \\ -4x_1 + 6x_2 & \leq 9 \\ x_1 + x_2 & \leq 4 \\ x_1, x_2 & \geq 0 \text{ integer} \end{aligned}$$

solve it via the cutting plane method with Gomory's fractional cutting planes.