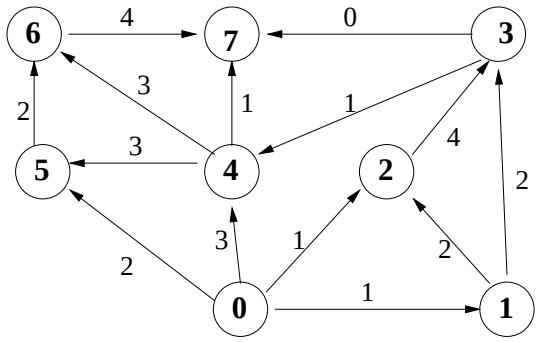
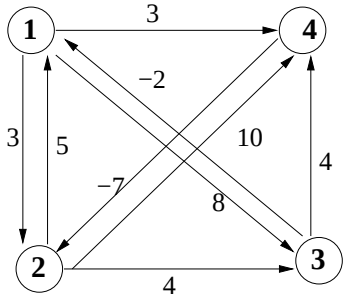
2.5 Shortest paths with nonnegative costs

Given the following directed graph, find a set of shortest paths from node 0 to all the other nodes, using Dijkstra's algorithm. Can we solve the problem with Dynamic Programming? If yes, do so.



2.6 Shortest paths with negative costs and ill-posedness

Given the following directed graph, find the shortest paths between all pairs of nodes, or show that the problem is ill-posed by exhibiting a circuit of total negative cost.



2.7 An application of Dynamic Programming to machine renewal

A company must buy a new machine and then determine a renewal (maintenance-replacement) plan for the next 5 years, making sure that, at any point in time, the available machine works properly. At the beginning of each year of the planning horizon, the company must decide whether to keep the old machine or to substitute it with a new machine.

The maintenance costs and the expected revenue (when the machine is sold) clearly depend on how old the machine is and they are indicated in the following table.

years	maintenance (kEuro)	revenue when sold (kEuro)
0	2	-
1	4	7
2	5	6
3	9	2
4	12	1

To avoid high maintenance costs of an old machine, the machine can be sold at the beginning of the second, third, fourth, and fifth year, and an new one can be bought. For the sake of simplicity, we suppose that a new machine always costs 12K Euro.

Show that the problem of determing a machine renewal plan of minimum total net cost (total cost for buying/rebuying the machine + maintenance costs - total revenue) can be solved via Dynamic Programming by finding a shortest path in an ad hoc directed acyclic graph. Find an optimal renewal plan. Is it unique?

2.8 Project planning

The preparation of the apple pie has long been a tradition at Rossi's family. First the weight of the ingredients has to be determined: flour, sugar, butter, eggs, apples, cream. The butter must then be melted down, and added to a mixture of flour, sugar, and eggs. Apples must be added to this new mixture, once they have been peeled and cut into thin slices. The mixture can then be cooked, in the already heated oven. It is advisable to whip the cream only after the apple slices have been added to the mixture. Once the cake is cooked, the cream is used to garnish it.

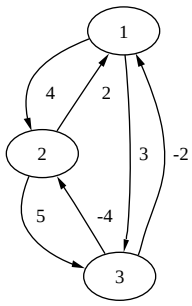
The following table reports the time needed for each activity.

Activity		Time (minutes)
A	Weight the ingredients	5
B	Melt the butter	3
C	Mix flour, eggs and sugar	5
D	Peel the apples and cut them into slices	10
E	Heat the oven	20
F	Add butter to the mixture	8
G	Add apples to the mixture	4
H	Cook the mixture in the oven	40
I	Whip the cream	10
L	Garnish	5

Draw the graph (with activities associated to arcs) which represents the project precedence relations. Determine the minimum total completion time of the project as well as the earliest times and latest times associated to each node. Identify the critical activities and draw Gantt's chart at earliest.

2.9 Shortest paths with negative costs

Given the following directed graph, find the shortest paths between all pairs of nodes, or show that the problem is ill-posed, by exhibiting a circuit of total negative cost.


1
2
3
4
2
3
-2
5
-4