### 2.10 Maximum flow and minimum cut

Given the following network with capacities on the arcs

find a maximum (feasible) flow from node 1 to node 7 , and determine a corresponding minimum (capacity) cut.

### 2.11 Maximum flow with node capacities

In maximum flow problems, how can we deal with capacities on both nodes and arcs? Find a maximum flow from node 1 to node 7 in the network of the previous exercise, with a node capacity of 2 on node 6 .

### 2.12 Maximum flow with a strictly positive initial feasible flow

Given the following network with capacities on the arcs

find a maximum flow from node 1 to node 7 , starting from the feasible flow of value 10 in which 10 units are sent along the path 1-3-6-5-4-7. Determine a corresponding minimum cut.

### 2.13 Indirect application of maximum flows

A software house has to handle 3 projects, $P_{1}, P_{2}, P_{3}$, over the next 4 months. The projects require, respectively, 8,10 , and 12 man-months. $P_{1}$ can only begin after month 1 , and must be
completed (at latest) by the end of month $3 . P_{2}$ and $P_{3}$ can begin from month 1 , and must be completed, respectively, by the end of month 4 and 2 , respectively. Each month, 8 engineers are available. Due to the internal structure of the company, no more than 6 engineers can work, at the same time, on the same project.

Determine whether it is possible to complete the three projects within the time constraints and, if it is possible, find a feasible workforce plan. Describe how this problem can be reduced to the problem of finding a maximum flow in an appropriate network.
[Hint: Look for a feasible flow of value 30?]

