

## 1.8 Modem production

A company A, producing a single type of modem, is planning the production for the next 4 months. The production capacity is of 2500 units per month, at a cost of 7 Euro per unit. A second company B can be payed to produce extra units, at a cost of 9 Euro per unit, for a maximum of 700 units per month.

Inventory can be used, at a cost of 1 Euro per month per unit. 70 units are stored at the beginning of the planning period, and 300 units must be stored at the end of the period.

For technical reasons, the units must be produced in lots of, at least, 80 units. Sales forecasts for the next 4 months indicate the following demand

Month	1	2	3	4
Demand	2600	3000	2350	2700

Give an Integer Linear Programming (ILP) formulation for the problem of determining a production plan of minimum total cost.

Suppose that fixed charges  $f_A$  and  $f_B$  must be payed each month that production takes place in factory A and, respectively, B. How can we extend the ILP formulation to account for fixed charges?

## 1.9 Hospital shifts

We have to plan the shifts for a hospital department, so that the total number of nurses is minimized. Each nurse works for 5 consecutive days, and stays home for the next two days. The estimated demand, in term of number of nurses, is

Day	Mo	Tu	We	Th	Fr	Sa	Su
Demand	11	9	7	12	13	8	5

Give an integer linear programming formulation for the problem.

## 1.10 Cash flow

A company expects the following cash flow (in kEuro) for the next 8 months

Month	inflow	outflow
May	100	500
June	200	500
July	200	600
...	...	...
Dec	900	100

At the beginning of the planning period (May), we have 100K Euro of cash. Money can be loaned from banks, with two options: 1) loan for 8 months, with 9% interest overall, or 2) loan for 1 month, with 1.5% interest per month. For instance, with option 1, if 100 Euro are loaned at the beginning of May, 109 Euro must be given back at the end of December; with option 2, if 100 Euro are loaned at the beginning of month  $i$ , 101.5 Euro must be given back at the beginning of month  $(i + 1)$ .

No debt must be established after the end of the planning period. A cash of 50k Euro is required at the end of the period.

Give a linear programming formulation for the problem of minimizing the loaning cost.

How should we modify the formulation if only one of the two loaning options can be chosen?