Foundations of Operations Research
Practice exercises: Linear Programming

M. Ciavotta

2018/2019

Exercise 1

Solve the following linear problem graphically:

$$\begin{aligned} \max \quad & 2x_1 + x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 10 \\ & x_1 \leq 4 \\ & x_2 \leq 5 \\ & x_1, x_2 \geq 0. \end{aligned}$$

Exercise 2

Solve the following linear problem graphically:

$$\begin{aligned} \min \quad & -x_1 - x_2 \\ \text{s.t.} \quad & \\ & -3x_1 + 2x_2 \geq 6 \\ & 3x_1 + x_2 \geq 9 \\ & x_1, x_2 \geq 0. \end{aligned}$$

Exercise 3

Determine using the Simplex algorithm with Bland's rule the optimal solution to the following linear programming problem:

$$\begin{aligned} \max \quad & x_1 + 3x_2 + 5x_3 + 2x_4 \\ \text{s.t.} \quad & x_1 + 2x_2 + 3x_3 + x_4 \leq 3 \\ & 2x_1 + x_2 + x_3 + 2x_4 \leq 4 \\ & x_1, x_2, x_3, x_4 \in \mathbb{R}^+. \end{aligned}$$

Exercise 4

Determine using the Simplex algorithm with Bland's rule the optimal solution to the following linear programming problem:

$$\begin{aligned} \min \quad & -5x_1 - 2x_2 - 3x_3 - x_4 \\ \text{s.t.} \quad & x_1 - 2x_2 + 2x_3 + 2x_4 \leq 4 \\ & -x_1 + x_2 + x_3 - x_4 \leq 6 \\ & x_i \geq 0. \end{aligned}$$

Exercise 5

Solve the following linear programming problem using the Simplex algorithm with Bland's rule:

$$\begin{aligned} \min \quad & 3x_1 + x_2 + x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 + x_3 = 6 \\ & x_1 + x_2 + 2x_3 = 2 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

Exercise 6

Consider the following linear programming problem:

$$\begin{aligned} \max \quad 2x_1 + x_2 \\ -2x_1 - x_2 \leq -1 \\ x_1 - x_2 \leq 3 \\ 4x_1 + x_2 \leq 17 \\ x_2 \leq 5 \\ -x_1 + x_2 \leq 4 \end{aligned}$$

where $x_1, x_2 \geq 0$.

a) Write the dual problem of the given problem.

b) Write the equations defining the complementarity slackness for the given problem (Notice that the problem and its dual are in symmetric form).

c) Exploiting the complementarity conditions say whether points (3,5) and (4,1) are optimal.

Exercise 7

Consider the following problem:

$$\max z = 9x_1 + 8x_2$$

$$\begin{aligned} x_1 - 2x_2 &\leq -1 \\ 4x_1 + 3x_2 &\leq 4 \\ -x_1 + 2x_2 &\leq 3 \\ 2x_1 - x_2 &\leq -4 \end{aligned}$$

Verify that solution $x_1 = -3$, $x_2 = -1$ and $x_1 = -\frac{5}{3}$, $x_2 = \frac{2}{3}$ are optimal.

Exercise 8

Consider the following Linear Programming problem:

$$\min -x_1 + 2x_2 \tag{1}$$
$$-x_1 + x_2 \leq 2 \tag{2}$$
$$x_1 + x_3 = 3 \tag{3}$$
$$2x_1 + x_2 \geq 1 \tag{4}$$
$$2x_1 - 6x_2 \leq 15 \tag{5}$$
$$x_1, x_3 \geq 0, \quad x_2 \text{ free} \tag{6}$$

Without applying any problem transformation, write the dual problem and the complementary slackness conditions (for both problems).

Consider two solutions $x^1 = \left[3, -\frac{3}{2}, 0\right]$ and $x^2 = \left[\frac{3}{2}, -2, \frac{3}{2}\right]$. Determine the dual complementary solutions and discuss the optimality of both primal solutions.

Exercise 9

Consider the following Linear Programming problem:

$$\begin{aligned} \min -2x_1 + 2x_2 - 2x_3 & \\ 2x_1 - 2x_2 - x_3 &\leq 2 \\ -3x_1 + 3x_2 + 2x_3 &\leq 3 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

- Write the dual problem.
- Solve the primal problem using the Simplex method with Bland's rule.